Assignment- 02.

Unit -2

Differential Calculas.

Que 1. Calculate $\frac{d^3y}{dx^3}$ if $y = 2\sin 2x + \cos x$

Solve.

$$\frac{dy}{dx} = 4\cos 2x - \sin x$$

$$\frac{d^2y}{dx^2} = -8\sin 2x - \cos x$$

$$\frac{d^3y}{dx^3} = -16\cos 2x + \sin x$$

Que 2. find the maximum and minimum Value of the function $f(x) = x^3+1$, $x \in R$.

Solve

Cubic function, which goes to $-\infty$ as $x \to -\infty$ and $+\infty$ as $x \to +\infty$

$$f'(x) = 3x^2$$

$$3x^2 = 0$$

$$x = 0$$

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Evaluating the Value at Critical Point

$f(0) = 0^3 + 1 = 1$

~~Minimum the Value at Critical Point.~~

$\cancel{f(0)} =$

Minimum Value : 1 at $x = 0$

No Maximum Value as $x \to +\infty$

$f(x) \to +\infty$

Ans.

Que 3. find the first four term in the expansion of $\log(1+\sin x)$ by Maclaurins Theorem.

Solve

$$f(x) = \log(1+\sin x)$$

$$f(0) = \log(1+\sin(0))$$

$$= \log(1+0)$$

$$f(0) = 0$$

$$f'(x) = \log(1+\sin x)$$

$$= \frac{1}{1+\sin x} \cdot \cos x$$

$$f'(x) = \frac{\cos x}{1+\sin x}$$

$$f'(0) = \frac{\cos 0}{1+\sin 0} = \frac{1}{1+0} = 1$$

$$f''(x) = \frac{1+\sin x \frac{d}{dx}\cos x - \cos x \frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{-(1+\sin x).(\sin x) - \cos x\,[\cos x]}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$= \frac{-[\sin x + 1]}{(1+\sin x)^2}$$

$$f''(x) = \frac{-1}{1+\sin x}$$

$$f''(0) = \frac{-1}{1+\sin(0)}$$

$$f'''(x) = \frac{\cos x}{(1+\sin x)^2}$$

$$f'''(0) = \frac{\cos(0)}{(1+\sin(0))^2} = \frac{1}{(1+0)^2} = 1$$

$$\log(1+\sin x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$$= 0 + x.1 + \frac{x^2}{2\times1}(-1) + \frac{x^3}{3\times2\times1}(1)$$

$$\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6}$$

Que 4. Analyse the three diffrent types of ~~Asympto~~ Asymptotes.

Ans

1. Vertical Asymptotes - Vertical Asymptotes occur where a function approaches infinity (or negative infinity) or the input approaches a specific Value.

2. Horizontal Asysmptotes - ~~Vertical~~ Horizontal Asymptotes indicate the behavior of a function as x approaches infinity (or negative infinity). They represent the Value that $f(x)$ approaches.

3. Oblique (slant) Asymptotes - Oblique Asymptotes occur when the degree of the numerator is exactly one greater than the degree of the denominator. These represent a linear function that the graph approaches.

Ques. Find the Taylor's Series expansion of the function $f(x) = \log(\cos x)$ about the Point $\pi/3$.

Solve

We know,

$$f(x) = f[x - (x-a)]$$

$$= f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \; f'''(a)$$

$$f(x) = \log(\cos x)$$

$$f(\pi/3) = \log \cos\frac{\pi}{3} = \log\frac{1}{2} = \log 1 - \log 2 = -\log 2$$

$$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$$

$$f'(\pi/3) = -\tan\frac{\pi}{3} = -\sqrt{3}$$

$$f''(x) = -\sec^2 x$$

$$f''(\pi/3) = -\sec^2(\pi/3) = -4$$

$$f'''(x) = -[2\sec x \cdot \sec x \tan x]$$

$$= -2\sec^2 x \tan x$$

$$f'''(\pi/3) = -2\sec^2(\pi/3)\tan(\pi/3)$$

$$= -2(4)\sqrt{3}$$

$$= -8\sqrt{3}$$

$$f(\pi/3 + x - \pi/3) = -\log 2 + (x - \pi/3)(-\sqrt{3}) +$$

$$+ \frac{(x-\pi/3)}{2!}(-4) + \frac{(x-\frac{\pi}{3})^3}{3!}(-8\sqrt{3})$$

$$= -\log 2 - \sqrt{3}\left(x - \frac{\pi}{3}\right) - 4^{2}\,\frac{\left(x - \frac{\pi}{3}\right)}{2\times 1} - \frac{\overset{4}{8}\sqrt{3}\left(x - \frac{\pi}{3}\right)^{3}}{3\times 2\times 1} + \ldots\ldots$$

$$= -\log 2 - \sqrt{3}\left(x - \frac{\pi}{3}\right) - 2\left(x - \frac{\pi}{3}\right) - \frac{4\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^{3} + \ldots$$

Ans.